### 

### Semester Two Examination, 2020

### Question/Answer booklet

# MATHEMATICS SPECIALIST

**UNITS 1&2**

## Section One:

## Calculator-free

Your Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Your Teacher’s Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Question | Mark | Max | Question | Mark | Max |
| 1 |  | 4 | 6 |  | 5 |
| 2 |  | 9 | 7 |  | 7 |
| 3 |  | 7 | 8 |  | 5 |
| 4 |  | 9 |  |  |  |
| 5 |  | 4 |  |  |  |

**Structure of this paper**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 50 | 36 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 100 | 64 |
|  |  |  |  | **Total** | 100 |

**Instructions to candidates**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.**Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

**See Next Page**

**Section One: Calculator-free (50 marks)**

This section has **8 (eight)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

● Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.

● Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

**Question 1 (2.3.1) (4 marks)**

Prove the following statement:

If and are odd integers, then is divisible by 4.

|  |
| --- |
| **Solution** |
| Assume that and are both odd integers.  Then and for some , and so  which is divisible by 4.  Hence is divisible by 4.  QED |
| **Specific behaviours** |
| 🗸 assumes and are odd  🗸 expresses as and as  🗸expands and factorises  🗸concludes that is divisible by 4 |

**Question 2 (2.2.1 - 2.2.3, 2.2.11) (9 marks)**

1. Let .
2. Determine .(2 marks)

|  |
| --- |
| **Solution** |
| Hence |
| **Specific behaviours** |
| 🗸 calculates determinant  🗸 writes correct expression for |

1. Showing use of an appropriate matrix equation together with your answer to part (i), determine the coordinates of the point of intersection of the lines

and .

(3 marks)

|  |
| --- |
| **Solution** |
| Lines intersect at where satisfies:  Now  Hence the lines intersect at |
| **Specific behaviours** |
| 🗸 writes correct matrix equation  🗸 multiplies both sides by  🗸 states coordinates of point of intersection |

1. Consider the equation

where .

1. Determine the value of such that the equation **does not** have a unique solution for. (2 marks)

|  |
| --- |
| **Solution** |
| No unique solution if , which is true if .  Hence if , the equation does not have a unique solution. |
| **Specific behaviours** |
| 🗸 equates determinant to 0  🗸 states |

1. With the value of obtained in part (i), what is the geometrical relationship between the lines and ? (2 marks)

|  |
| --- |
| **Solution** |
| With , the first equation is times the second. Hence the lines are coincident. |
| **Specific behaviours** |
| 🗸 states that equations are scalar multiples of each other  🗸 states that lines are coincident/same |

**Question 3 (2.3.4, 2.3.5) (7 marks)**

Use the principle of mathematical induction to prove that

for all integers .

|  |
| --- |
| **Solution** |
| Let stand for the statement ‘’ for all .  In :  Hence LHS = RHS and so is true.  Now assume that is true for some integer .  Then .  Now  This shows that is also true.  Hence, by PMI, is true for all integers . |
| **Specific behaviours** |
| 🗸 defines  🗸 shows that is true by evaluating LHS and RHS separately  🗸 assumes is true  🗸 writes LHS of in terms of RHS of  🗸 simplifies expression to obtain  🗸 concludes that is also true  🗸 concludes proof by referring to PMI |

**Question 4 (2.3.7-2.3.10) (9 marks)**

Let and . Write each of the following in the form where .

1. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 adds real and imaginary parts separately  🗸 states |

1. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 expands correctly  🗸 states |

1. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 determines conjugate of or uses  🗸 states |

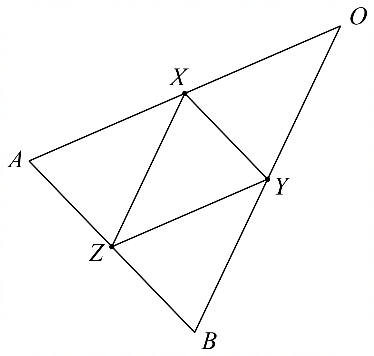
1. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 multiplies numerator and denominator by  🗸 expands  🗸 states (must be in form i.e. don’t accept as final answer) |

**Question 5 (1.2.11, 1.2.12) (4 marks)**

Let be an isosceles triangle with , and let and be the midpoints of , and respectively. Let and .

Use a vector method to prove that is isosceles.



|  |
| --- |
| **Solution** |
| Thus and .  But since is isosceles, , and hence , meaning that is isosceles. |
| **Specific behaviours** |
| 🗸 writes as  🗸 shows that  🗸 shows that  🗸 deduces from that |

**Question 6 (2.1.3 - 2.1.6, 2.1.8) (5 marks)**

Prove that the following is true for all .

|  |
| --- |
| **Solution** |
| Hence for all . |
| **Specific behaviours** |
| 🗸 writes cot expressions using  🗸 combines to give a single fraction  🗸 using double angle formula for cos  🗸 simplifies to  🗸 works from LHS to RHS (or vice versa) |

**Question 7 (1.3.2, 1.3.5, 2.3.1) (7 marks)**

1. Let be an irrational number and a rational number. Use the method of proof by contradiction to prove that is irrational. (4 marks)

|  |
| --- |
| **Solution** |
| Assume that is irrational and is rational, but that is rational.  Then and for some integers , , and .  Now  which is rational since and are integers. This contradicts the assumption that is irrational; hence must be irrational. |
| **Specific behaviours** |
| 🗸 assumes that is rational  🗸 writes and as a ratio of integers  🗸 shows that is rational  🗸 notes contradiction and concludes that is irrational |

1. State whether the following is true or false and prove or disprove it accordingly:

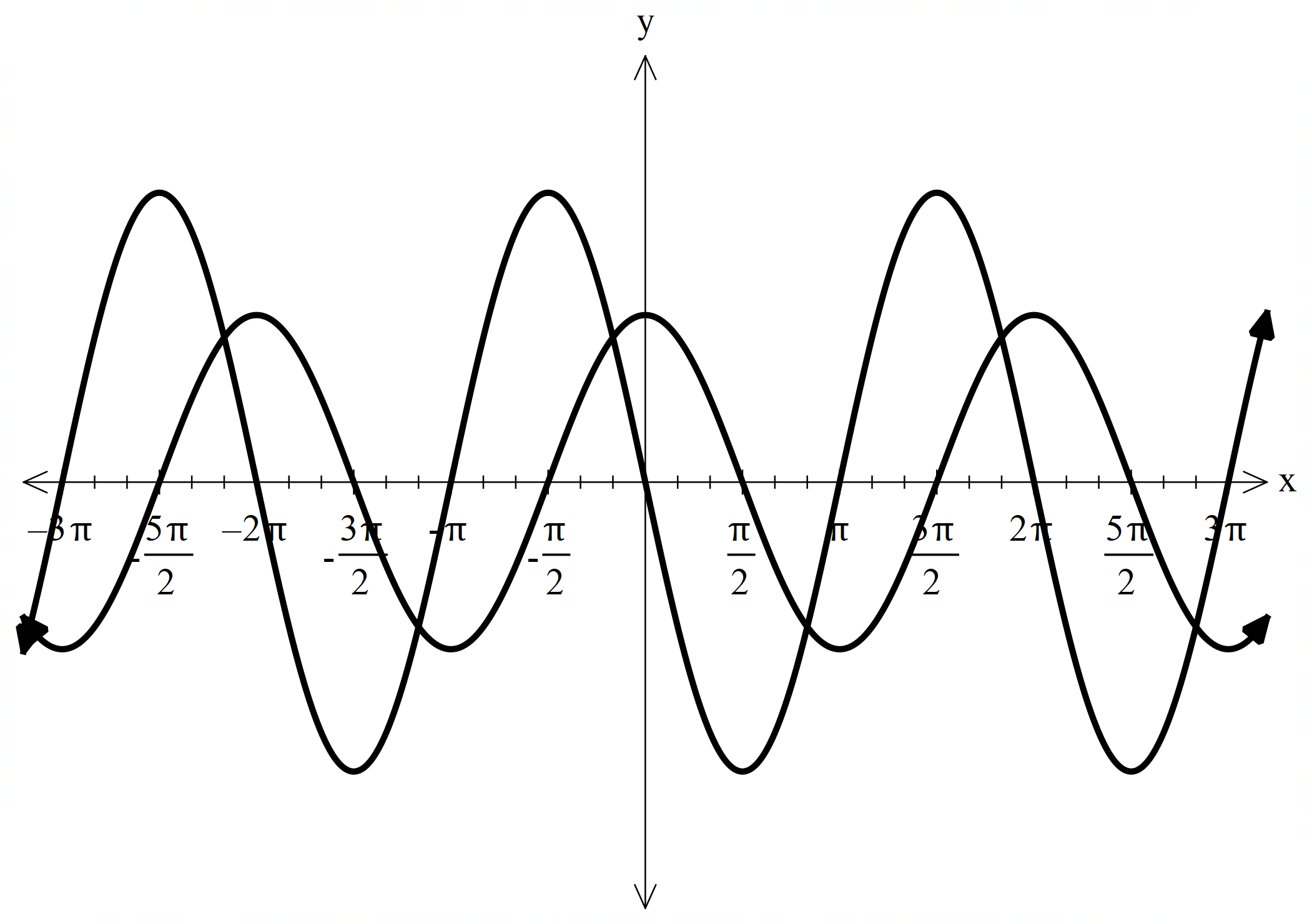
‘If is irrational and is rational, then is irrational.’

(3 marks)

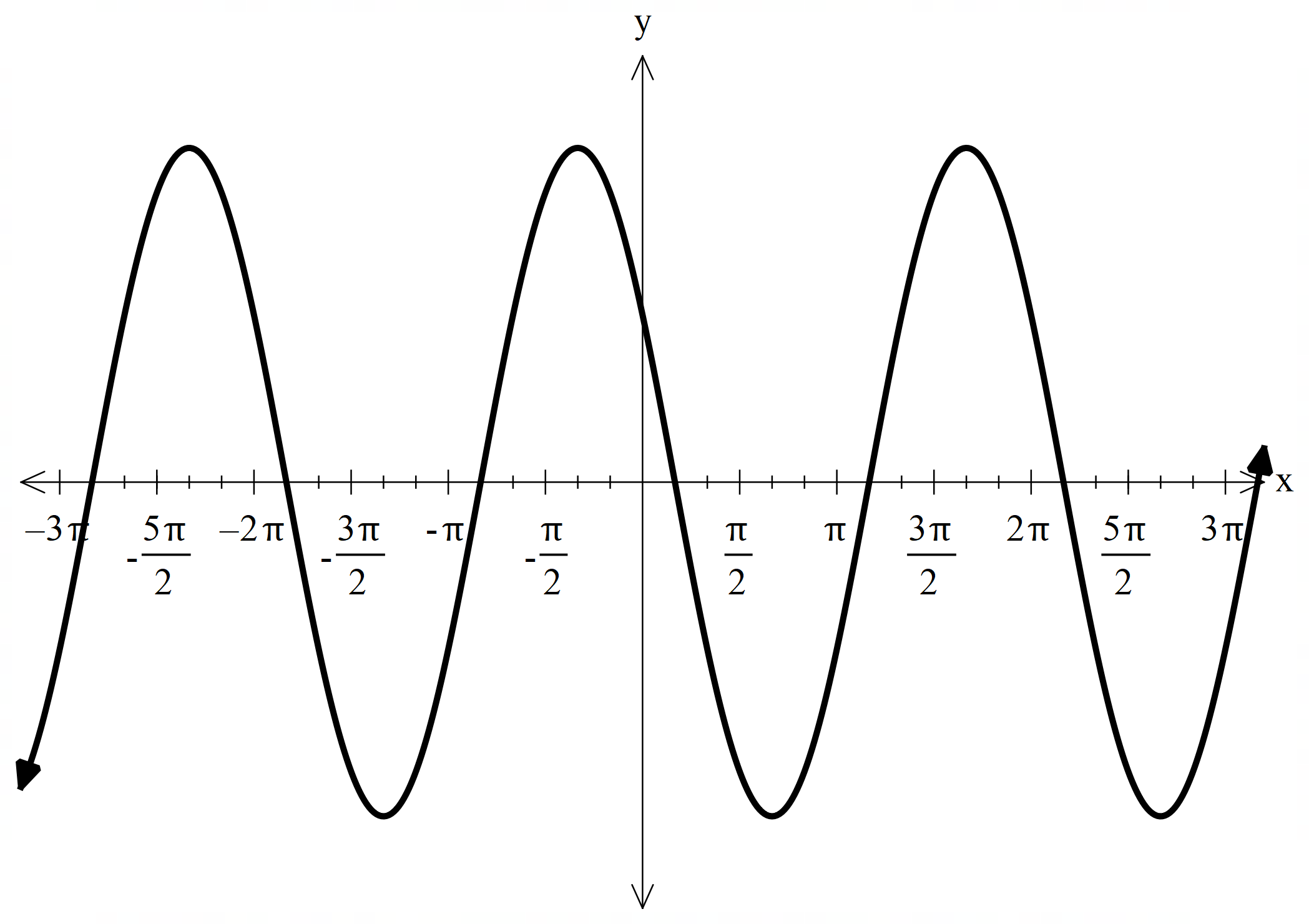
|  |
| --- |
| **Solution** |
| The statement is false.  E.g. let and . Then is irrational and is rational, but , which is rational. |
| **Specific behaviours** |
| 🗸 states false  🗸 gives counterexample values for and  🗸 shows that is rational for those values |

**Question 8 (2.1.7) (5 marks)**

The diagram below shows the graphs of two functions and where is a real constant.



Below is the graph of , which can also be expressed as where and are positive real constants.



1. Determine an exact value for . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 states correct value |

1. Determine thevalues of and . (4 marks)

|  |
| --- |
| **Solution** |
| Equation of graph is  Since the equation of the graph is also , we have , so .  Hence . |
| **Specific behaviours** |
| 🗸 notes that  🗸 expands using compound angle formula  🗸 solves for and states correct value  🗸 solves for and states correct value |

**END OF SECTION ONE**

Additional working space

Question number:

Additional working space

Question number: